TURBULENT MIXING OF ACCOMPANYING AND OPPOSING FLOWS

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The solution of the problem of the edge of a turbulent jet, obtained in [1] starting from a model of the vortical motion of an ideal liquid, is generalized for the case of the turbulent mixing of two plane semibounded (accompanying or opposing) flows of an incompressible liquid. It is shown that the results of calculation are in qualitative agreement with experimental data and that they are close to them quantitatively. Some of the special characteristics of the method are discussed.

1. In [1] an analytical solution is given to the problem of the two-dimensional mixing of a plane homogeneous stream and a stationary liquid (the problem of the "edge of the jet" in the theory of turbulent jets [2]). The starting point for construction of the theory in [1] was a model of the vortical motion of an ideal (nonviscous) incompressible liquid.

The analogy between this type of motion and fully developed (established, on the average) turbulent flow has been known for a long time ([3], page 271).

In [1] the authors do not leave out of consideration the question of the origin of vortices and treat it on the basis of a model of the mixing layer forming with the decomposition of an unstable tangential discontinuity of the velocity into a large number of individual vortices. By averaging in time of the actual values of the velocity of the unsteady-state potential flow of a liquid, in which isolated vortices are moving, the authors impart to the model the principal properties of average turbulent motion, particularly "losses of memory" with respect to the initial state. This method, i.e., a transition to an averaged flow, which is independent of the initial random conditions, is an approximation on the road to a direct calculation of turbulent motion by the solution and subsequent averaging of the unsteady-state Navier–Stokes equations. The model adopted can reflect three characteristic negative criteria of the actual turbulent motion, i.e., its unsteady-state character, its nonlinearity, and its irregularity, and can confer typical statistical properties on the averaged flow, i.e., steady-state conditions, continuity, and an ordered character.

Neglect of the effect of viscosity is admissible for fully developed turbulent motion far from solid walls and for the region of scales in which viscous dissipation is insignificant.

The solution of the problem obtained in [1] and its calculated examples, i.e., the profiles of the longitudinal component of the velocity and its dispersion, bear witness to qualitative agreement with experiment. The results of calculation are quantitatively close to the experimental values (on the order of magnitude of the value of the pulsations of the velocity $\langle u'^2 \rangle$, of the value of the relative velocity $\langle u \rangle / u_1 \approx 0.7$ at the prolongation of the line of separation, and of others). It is of interest to make a more complete examination of the proposed model and to make a comparison of the results of calculations of the solution and experimental data, and, specifically, to supplement the calculation of the longitudinal component of the velocity by calculation of the transverse (mean and pulsational) component of the velocity, and to determine the value of the turbulent friction stress inherent in the model and the values of the correlations and their distributions in the field of the flow.

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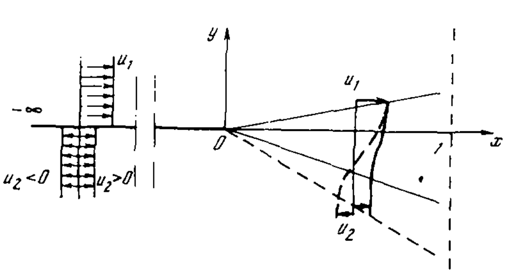

Fig. 1

We shall carry this out using the example of the problem of the free turbulent mixing of two homogeneous, plane-parallel, accompanying or opposing flows of an incompressible liquid [2]. A partial case of this problem is the problem of the edge of a jet [1]. A consideration of accompanying flows permits making a comparison between the model and experiment, from a detailed study in the experiments of the effect of the parameter of accompaniment m [the ratio of the velocities of the flows $m = u_2/u_1$ (see Fig. 1)] on the characteristics of the mixing layer. Generalization of the problem for opposing flows is of interest from the point of view of a comparison between calculation and approximate results obtained within the framework of the theory of the boundary layer, i.e., for parabolic equations [2].

2. Schematically (Fig. 1), the problem of the turbulent mixing of accompanying flows or opposing flows reduces to the following. On both sides of a thin semiinfinite plate, two homogeneous plane-parallel flows of an incompressible liquid move at different velocities. We neglect the friction of the liquid on the plate, i.e., we shall not take account of the effect of the boundary layer near the wall near the surface. We superpose the plate on the x axis in the region $x < 0$ and its edge on the origin of coordinates. Starting from this point ($x = 0$, $y = 0$), the separate flows (for definiteness, we shall speak of an accompanying flow) come into contact. For the free flow of a nonviscous liquid, under the conditions of the problem there is no characteristic dimension. Therefore, we must expect, and this is confirmed experimentally for problems of free turbulence, self-similarity of the fully developed averaged flow. This means that the relative values of the averaged valocity

$$\frac{\langle u\rangle}{u_1} = f\left(\frac{y}{x}, m\right) \quad \left(\langle u\rangle = \frac{1}{\Delta t}\int\limits_{t-\Delta t/2}^{t+\Delta t/2} u(t)\,dt\right)$$

are universal and depend on the ratio of the coordinates y/x (but not on y and x individually).

For a mathematical statement of the problem, we formulate the boundary conditions. We shall assume that the values of the velocity of the flows at the boundaries of the region are given by

$$u = u_1 \text{ with } x \to -\infty,\ y > 0,\ -\infty < x < +\infty,\ y \to +\infty$$
$$u = u_2 \text{ with } x \to -\infty,\ y < 0,\ -\infty < x < +\infty,\ y \to -\infty$$

We give a discontinuity of the velocity at the lines of the flow $y = 0$, simulating in the left-hand halfplane the plate:

$$u(x, +0) - u(x, -0) = u_1 - u_2,\quad v(x, +0) - v(x, -0) = 0$$

As a condition closing the calculating region we assume, following [1], that, at an arbitrary boundary of the flow at a distance L from the edge of the plate, the flow is parallel to the x axis, i.e.,

$$v(x, y, t) = 0 \text{ with } x = L,\quad -\infty < y < +\infty$$

The artificial limitation of the length of the mixing region is connected with the inclusion of the scale of length L, which is lacking in the physical statement, in the conditions of the problem. This leads to an imposed solution for the character of the flow near this boundary with $x \to L$ which is unreal for the physical problem. In actuality, at any arbitrary distance from the edge of the plate differing from zero, not only the actual values of the component $v(x, y, t) \neq 0$, but also the averaged value $\langle v\rangle(x, y) \neq 0$. Therefore, the best agreement with experiment is to be expected from a solution near the plate, i.e., with $x \ll L$. Self-similar flow should correspond to a limiting solution with $x/L \to 0$. With $x/L \to 1$, there is inevitably an appreciable difference between the calculated results and the real properties of the flow. Since $\langle v\rangle \ll \langle u\rangle$, this effect of the boundary condition with $x = L$ can distort only slightly the profile of the longitudinal component of the averaged velocity in the region $x \ll L$. The effect can be considerable for the profile of the transverse component (mean and pulsational) as well as for the correlation and the Reynolds stress (in [1] the profiles of $\langle u\rangle$ and $\langle u'^2\rangle$ are given only for the single cross section $x/L = 0.5$).

3. The postulation of the distribution of the vorticity

$$\omega_z \equiv \omega = \partial v / \partial x - \partial u / \partial y \tag{3.1}$$

in the field of the flow is essential. Following [1], we shall assume a value $\omega = 0$ in the whole mixing region, except for points at which individual vortices, generated with the decomposition of the interface, are are located at any given moment of time. Introducing the flow function in accordance with the formulas

$$u = \partial \psi / \partial y, \qquad v = -\partial \psi / \partial x$$

we connect in with the vorticity using the Poisson equation

$$\Delta \psi = -\omega \tag{3.2}$$

The time-averaged circulation of the vector of the velocity in the mixing region is made up of the circulation created by the boundary discontinuity of the velocity with $x < 0$

$$\Gamma_{x<0} = \int_{-\infty}^{0} \int_{y=-0}^{y=+0} (u_2 - u_1)\, dx dy$$

and the total circulation of the system of N point vortices, located on the average (after the time Δt) in the band $0 < x < L$,

$$\Gamma_L = N\Gamma_1 = \frac{1}{\Delta t} \int_{t-\Delta t/2}^{t+\Delta t/2} \int_{0}^{L} \int_{-\infty}^{\infty} \omega(x, y, t)\, dt dx dy = (u_2 - u_1) L$$

Going over from the circulation Γ to the vorticity ω, we reduce the problem to the integrated Poisson equation

$$\Delta \psi = (u_1 - u_2)\, \delta(y)\, \sigma(-x) + \frac{u_1 - u_2}{N} L \sum_{n=1}^{N} \delta[y - y_n(t)]\, \delta[x - x_n(t)] \tag{3.3}$$

where $\delta(z)$ and $\sigma(z)$ are symbols of a δ-function and a unit power function $[d\sigma(z)/dz = \delta(z)]$; $y_n(t)$ and $x_n(t)$ are the instantaneous coordinates of the n-th point vortex ($n = 1, 2, \ldots, N$). We give the solution of Eq. (3.3) without taking account of point vortices, i.e., for the steady-state problem

$$\Delta \psi_s = (u_1 - u_2)\, \delta(y)\, \sigma(-x) \quad (\psi_s = \psi_s(x, y)) \tag{3.4}$$

with the boundary conditions

$$\begin{aligned} &\partial \psi_s / \partial y = u_1 \text{ with } y \to \infty, \quad \partial \psi_s / \partial y = u_2 \text{ with } y \to -\infty \\ &\partial \psi_s / \partial x = 0 \text{ with } x \to -\infty, \quad \partial \psi_s / \partial x = 0 \text{ with } x = L \end{aligned} \tag{3.5}$$

By the substitution

$$\psi_s^* = \psi_s - y \left(\frac{u_1 - u_2}{2} + \frac{u_1 - u_2}{2} \operatorname{sign} y \right) \tag{3.6}$$

we reduce the problem to a Neumann problem with null values of the derivatives along a normal to the boundary at the boundaries of the calculating region

$$\Delta \psi_s^* = (u_1 - u_2)\, \delta(y)\, [\sigma(-x) - 1] \tag{3.7}$$

The boundary conditions for Eq. (3.7) are

$$\partial \psi_s^* / \partial y = 0 \text{ with } y \to \pm\infty, \quad \partial \psi_s^* / \partial x = 0 \text{ with } x \to -\infty, \quad x = L \tag{3.8}$$

The Green function for this problem has the form

$$G(x, y, x', y') = -\tfrac{1}{2} \ln [(x - x')^2 + (y - y')^2] - \tfrac{1}{2} \ln [(x + x' - L)^2 + (y - y')^2] \tag{3.9}$$

The solution of the Poisson equation (3.7) is expressed in terms of the Green function (3.9) by

$$\psi_s^*(x', y') = -\frac{1}{2\pi}\int_{-\infty}^{L}\int_{-\infty}^{+\infty} \Delta\psi_s^* G(x, y, x', y')\, dx dy = \frac{1}{2\pi}\int_{0}^{L} (u_1 - u_2)\, G(x, 0, x', y')\, dx \tag{3.10}$$

we obtain

$$\psi_s^*(x', y') = -\frac{u_1 - u_2}{4\pi}\left\{x' \ln(x'^2 + y'^2) - (x' - 2L)\ln[(x' - 2L)^2 + y'^2] - 4L + 2y'\left(\operatorname{arc\,tg}\frac{x'}{y'} - \operatorname{arc\,tg}\frac{x' - 2L}{y'}\right)\right\} \tag{3.11}$$

Taking account of point vortices, the solution of the complete equation (3.3) has the form

$$\psi(x', y', t) = y'\left(\frac{u_1 + u_2}{2} + \frac{u_1 - u_2}{2}\operatorname{sign} y'\right) + \psi_s^*(x', y') +$$
$$+ \frac{(u_1 - u_2)L}{4\pi N}\sum_{n=1}^{N}\{\ln[(x' - x_n)^2 + (y' - y_n)^2] + \ln[(x' - 2L + x_n)^2 + (y' - y_n)^2]\} \tag{3.12}$$

We go over to dimensionless variables and parameters, using the quantities L and u_1 as the scales of the length and the velocity:

$$x^\circ = \frac{x}{L}, \quad y^\circ = \frac{y}{L}, \quad t^\circ = \frac{t}{L/u_1}, u^\circ = \frac{u}{u_1}, \quad v^\circ = \frac{v}{u_1},$$
$$m = \frac{u_2}{u_1}, \quad \psi^\circ = \frac{\psi}{u_1 L}$$

(in what follows we shall omit the degree signs on the dimensionless quantities and the primes on the coordinates x' and y').

For the dimensionless components of the velocity, from solution (3.12) we have

$$u = \frac{\partial\psi}{\partial y} = \frac{1 + m}{2} + \frac{1 - m}{2}\operatorname{sign} y - \frac{1 - m}{2\pi}\left(\operatorname{arc\,tg}\frac{x}{y} + \right.$$
$$\left. + \operatorname{arc\,tg}\frac{-x + 2}{y}\right) + \frac{1 - m}{2\pi N}\sum_{n=1}^{N}(y - y_n)\left[\frac{1}{(x - x_n)^2 + (y - y_n)^2} + \frac{1}{(x - 2 + x_n)^2 + (y - y_n)^2}\right] \tag{3.13}$$

$$v = -\frac{\partial\psi}{\partial x} = \frac{1 - m}{4\pi}\ln\frac{x^2 + y^2}{(x - 2)^2 + y^2} - \frac{1 - m}{2\pi N}\sum_{n=1}^{N}\left[\frac{x - x_n}{(x - x_n)^2 + (y - y_n)^2} + \frac{x - 2 + x_n}{(x - 2 + x_n)^2 + (y - y_n)^2}\right] \tag{3.14}$$

These formulas determine the unsteady-state field of the velocities for the region under consideration if the functions $x_n(t)$ and $y_n(t)$ are known.

Let us write the equations for determining $x_n(t)$ and $y_n(t)$, i.e., the equations of motion of the individual vortices. For an ideal liquid, from the equations of motion there follows the equation for the transfer (conservation) of vorticity

$$d\omega / dt = \partial\omega / \partial t + u\partial\omega / \partial x + v\partial\omega / \partial y = 0$$

Since, for point vortices

$$\omega = \sum_{s=1}^{N}\omega_s = c\sum_{s=1}^{N}\delta[x - x_s(t)]\,\delta[y - y_s(t)]$$

we then arrive at the following expression:

$$\sum_{s=1}^{N}\delta[x - x_s(t)]\frac{\partial\delta[y - y_s(t)]}{\partial y}\left(v - \frac{dy_s}{dt}\right) + \sum_{s=1}^{N}\delta[y - y_s(t)]\frac{\partial\delta[x - x_s(t)]}{\partial x}\left(u - \frac{dx_s}{dt}\right) = 0$$

It can be seen that with $x \to x_s(t)$ and $y \to y_s(t)$

$$dx_s / dt = u(x_s, y_s, t), \quad dy_s / dt = v(x_s, y_s, t) \tag{3.15}$$

These equalities constitute evidence of the displacement of the vortices along with the flow of liquid. The coordinates of the point vortices $x_s(t)$ and $y_s(t)$ can be found from Eqs. (3.15), the right-hand parts of which are expressed by formulas (3.13) and (3.14) with the replacement of x and y by x_s and y_s. (In the summation, terms with n = s must be omitted, so as not to take account of the action of a vortex on itself.)

Before passing on to a description of a numerical calculation using Eqs. (3.13)-(3.15), we must make a few comments on the field of the velocity generated by the steady-state vortical field of the plate, i.e., by the boundary discontinuity of the velocity with $x < 0$. The steady-state part of the solution (ψ_s) corresponds to a profile of the longitudinal component of the velocity which is symmetrical with respect to the y axis [with a value $u = (u_1 + u_2)/2$ at the axis]. The transverse component with $y = 0$ differs from zero in the region $x < 0$. For the steady-state part of the solution the plate is not a line of flow. Calculation shows that, taking account of the unsteady-state part of the flow, i.e., of the velocity field induced by the vortices, the averaged flow practically satisfies the boundary condition $v = 0$ with $y = 0$, $x < 0$. The profile of the velocity $\langle u\rangle$ takes on the asymmetry characteristic for the problem under consideration ($\langle u\rangle \approx 0.7u_1$ with $y = 0$ and $m = 0$).

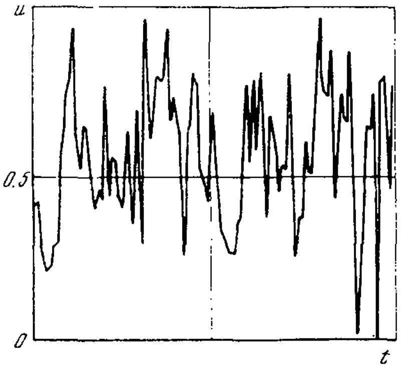

Fig. 2

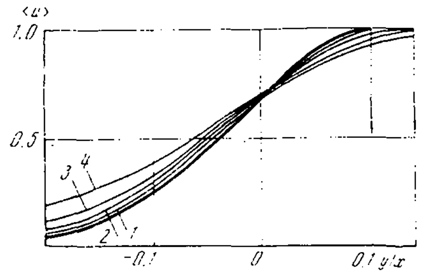

Fig. 3

4. Equations (3.13)-(3.15) were solved numerically in a BÉSM-6 digital computer for values of the parameter $m = u_2/u_1 = 0, 0.1, 0.25, 0.5$ as well as $m = -0.2$. In the calculation, a mean value of the number of vortices was taken, N = 50, and for purposes of comparison, N = 25 and 100. With a change in N, there was a variation of the time required for the establishment of steady-state conditions in an average field (approximately proportional to the number N).

The interval of time between the introduction of individual vortices τ with $m > 0$, taking account of the time that a vortex is located in the system $t_0 = 2/(1 + m)$, was assumed equal to t_0/N. With $m < 0$, the value was determined by a control calculation in such a way that the mean number of vortices in the region of the flow remained equal to a given value (for $m = -0.2$ the interval $\tau \approx 1.1\, t_0/N$). In the initial state N vortices are disposed along the line

$$y \approx 0 \quad \left(x_n = \frac{n-0.5}{N}, \quad y_n = 0.01 \sin\left(\frac{\pi}{2} x_n\right), \quad n = 1, 2, \ldots, N\right)$$

The choice of the initial velocity field had no effect on the averaged flow since, after a time equal to 50-100 τ, the pulsations of the velocity take on a random character. This can be seen from the example of a calculated oscillogram of the longitudinal component of the velocity at the point x = 0.5, y = 0.005 (Fig. 2, m = 0). After this time, the pulsation characteristics of the flow

$$\sigma_u = \langle u'^2\rangle^{1/2},$$
$$\sigma_v = \langle v'^2\rangle^{1/2},$$
$$\langle u'v'\rangle,$$
$$K_{uv} = \langle u'v'\rangle / \sigma_u\sigma_v$$

are found to be only approximately fully developed and can be used mostly for qualitative evaluations and for comparison with experiment.

5. Let us consider the question of the self-similar character of the flow for m = 0. Figure 3 gives four profiles of the longitudinal component of $\langle u\rangle$ in the calculating cross sections x = 0.2, 0.3, 0.4, and 0.5 (curves 1, 2, 3, 4) as a function of the ratio y/x. As can be seen from the figure, for different distances from the edge of the plate, the profiles of $\langle u\rangle$ differ appreciably among themselves. The difference increases with an increase in x and is a result of the artificial limiting condition with x = 1 (see above). The results obtained permit extrapolating the profile for the limiting value $x \to 0$. The self-similar profile of the velocity, shown by the heavy line in Fig. 3, will be used for comparison with the results of a calculation made using a semiempirical scheme which are in good agreement with experiment. The profile of the mean velocity selected using extrapolation is close to the calculated value with x = 0.2. In this cross sec-

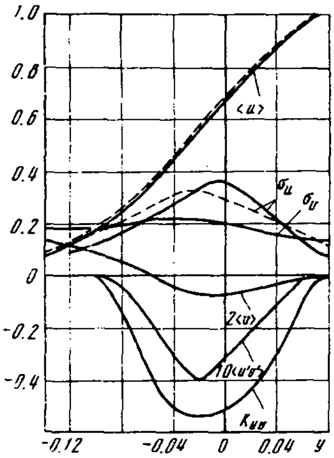

Fig. 4

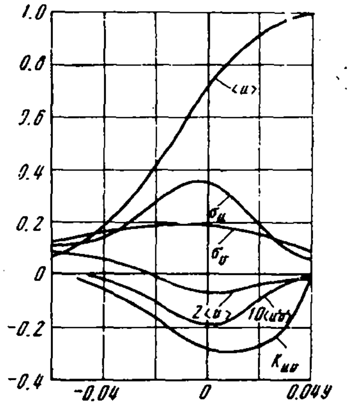

Fig. 5

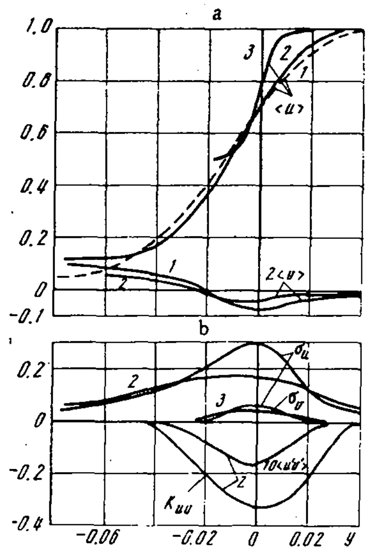

Fig. 6

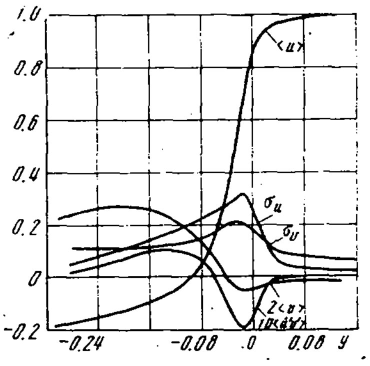

Fig. 7

tion, nearness to the "source of the vortices" has an effect on calculation of the pulsations. Therefore, in the succeeding figures for m = 0 and with analysis of data for m $\neq$ 0, the calculated results are given for the cross section x = 0.3 (with the exception of Fig. 4, in which the results of the calculation are compared with the data of [1]).

Figure 4 gives calculated profiles of the averaged and pulsation characteristics for m = 0 and x = 0.5. The dotted line shows the results of a calculation of ⟨u⟩ and σ_u carried out in [1], which are in agreement with those obtained in the present work. The remaining profiles (of the transverse component of ⟨v⟩ as well as of ⟨u'v'⟩, σ_v, and K_{uv}) have a form typical for the problem of the edge of a jet and, in order of magnitude, coincide with the experimental data of [2, 4-6]. During the course of the solution of the unsteady-state problem, there develops a stable correlation between the actual values of u' and v'; the maximal value of $K_{uv} \approx -0.5$. Supplementing the results of [1] for the edge of a jet, there is confirmed the possibility of an analytical determination of a quantity characteristic of turbulent flows, analogous to the Reynolds stress ($-\tau_T/\rho$ = ⟨u'v'⟩) as well as of the distribution of the transverse component of ⟨v⟩ (a value on the order of magnitude of $10^{-2}u_1$). In order of magnitude, the values of the pulsations of σ_u and σ_v attain 20-30% of u_1, which is in agreement with experiment.

Analogous profiles for x = 0.3 are shown in Fig. 5. In their character they are similar to the curves for x = 0.5 but, as a result of the great distance from the cross section x = 1, they are closer to self-similar.

Figure 6 shows profiles of the mean (a) and pulsational (b) characteristics for x = 0.3, with values of the parameter of accompaniment m = 0, 0.1, 0.5 (curves 1, 2, 3, respectively). It can be seen from the figure that the superposition of an accompanying flow leads to a constriction of the mixing region and to a lowering of the intensity of the pulsations. These properties are characteristic for real flows [2, 4-6], i.e., the results of calculation are in agreement with experiment.

Figure 7 gives analogous profiles of the mean and pulsational characteristics for an opposing flow (m = −0.2, x = 0.3). The solution makes it possible to obtain a qualitatively plausible picture of the motion, which is in agreement with calculations using a semiempirical scheme [2, 4, 5]. A quantitative comparison with experiment is difficult for this problem, due to the lack of detailed experimental data. There is a change in the sign of ⟨u'v'⟩ in Fig. 7 in the region of the transition to opposing flow (⟨u⟩ ⟨0⟩). The values of the pulsations are greater with m = −0.2 than with m = 0.2, i.e., with different absolute values of m, opposing flow corresponds to more intense turbulence.

To make a comparison between the calculated results and experiment, instead of experimental data differing among themselves, it is expedient to have recourse to the calculating formulas of semiempirical

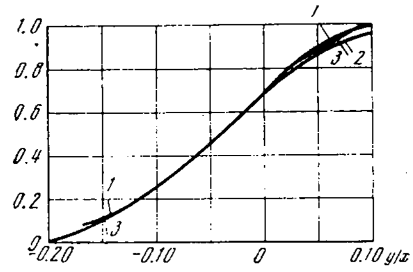

Fig. 8

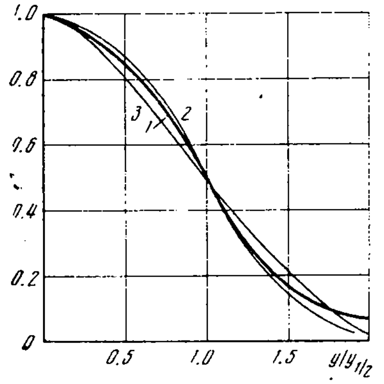

Fig. 9

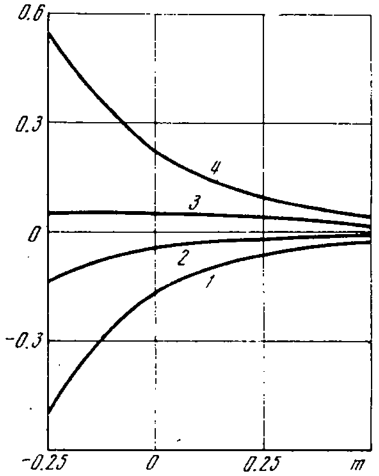

Fig. 10

theories. Figure 8, curve 1, gives a calculated profile of the velocity (the self-similar curve in Fig. 3, m = 0), curve 2 a profile of the velocity using a method equivalent to the problem of the theory of thermal conductivity [7], and curve 3 the so-called Schlichting profile [4]. (For curve 2

$$\langle u \rangle = \frac{1}{\sqrt{2}}\left[1 + \operatorname{erf}\left(\frac{y}{ax}\right)\right]^{1/2}$$

it is assumed that $a = 0.091$, starting from the agreement between the values of y/x with $\langle u \rangle = 0.1$. For curve 3 $\langle u \rangle = [1 - (y_s/b^{3/2}]^2$, for agreement it is assumed that $\langle u \rangle = 0.7$ with $y/x = 0$ and $y_s/b = 0.29$ and that $\langle u \rangle = 0.1$ with $y/x = -0.15$ and $y_s/b = 0.77$; this corresponds to $c \approx 0.33$ and to the equality $b = cx$.) As can be seen from Fig. 8, the calculated and semiempirical curves are close; the divergences lie within the limits of the usual scatter of the points.

Figure 9 in the coordinates

$$\hat{u} = \frac{\langle u \rangle - u_2}{u_1 - u_2}, \qquad \hat{y} = \frac{y}{y_{1/2}}$$

where $y_{1/2}$ is the coordinate of the point where $\hat{u} = 1/2$, gives a plot of the approximately universal profile (1) $\hat{u} = f(\hat{y})$ for all values of m ($-0.2 \leq m \leq 0.5$) and curves 2 and 3 analogously to Fig. 8. As curve 1 the mean profile from the distributions of the velocity for different values of m is taken. Curves 2 and 3 were recalculated from the expressions given in [7] and from the Schlichting profile.

In spite of the complexity of the comparison (superposition of the curves at the points $\hat{u} = 0.1$ and 0.7), it constitutes evidence of the fact that a solution free of the inclusion of empirical constants is in agreement with the results of a calculation using semiempirical schemes which have been confirmed experimentally. The solution corresponds to experiment not only qualitatively but, with a certain degree of approximation, also quantitatively. This conclusion can be extended also to opposing flows. It is of interest to clarify the effect of the parameter m on the geometry of the flow, i.e., on the dimensions and location (with respect to the line y = 0) of the mixing region.

Figure 10 illustrates the dependence on m of the geometric characteristics of the flow, i.e., of the relative coordinates y/x corresponding to the values $\hat{u} = 0.1$, 0.5, 0.9 (curves 1, 2, 3) and of the nominal thickness of the mixing zone $\Delta(y/x) = (y/x)_{0.9} - (y/x)_{0.1}$ (curve 4).

The figure shows that, in an accompanying flow ($m > 0$), the internal boundary of the flow (3) varies only slightly, while the external boundary (1) appreciably approaches the line y = 0 with a rise in the value of m. In the region $m < 0$, for opposing flows this boundary, with a rise in the absolute value of m, moves away from the straight line y = 0. The thickness of the mixing region $\Delta(y/x)$ falls continuously with a rise in the value of m. The graph in Fig. 10 shows that the empirical constants introduced into the calculating schemes of [2, 4, 5] depend on m.

6. What has been said confirms the applicability of the method of calculation proposed in [1] to the type of free turbulent flow under consideration. The field of the averaged characteristics of the flow ob-

tained by analytical solution (the components of the velocity and the secondary moments) corresponds to fully developed turbulent motion. The agreement between the solution and experiment was obtained from a two-dimensional model of the vortical flow of an ideal liquid, while the real actual motion is three-dimensional. Qualitatively, this can be explained by the fact that in jet turbulent flows, as is shown by experiment [2, 6], in practice, the secondary moments $\langle u'w'\rangle \approx \langle v'w'\rangle \approx 0$, containing the pulsations of the component of the velocity w along the z axis, are equal to zero. The effect of this component on the averaged motion may be negligibly small.

A further broadening of the method of [1] may be found useful in the solution of certain problems.

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